National Institute of Standards and Technology
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INFORMATION TECHNOLOGY LABORATORY

Network Formation by Contagion Averse Agents: Modeling Bounded Rationality with Logit Learning

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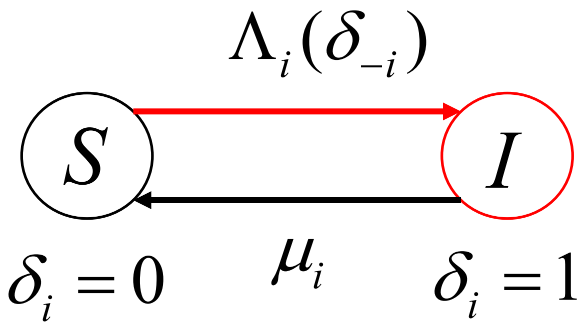
Susceptible Infected Susceptible (SIS) Model



$$\Lambda_i(\delta_{-i}) = \lambda \sum_{j \neq i} A_{ij} \delta_j$$

$A_{ij} = 0,1$ adjacency matrix

$\delta(t) = (\delta_i(t))$ Markov process

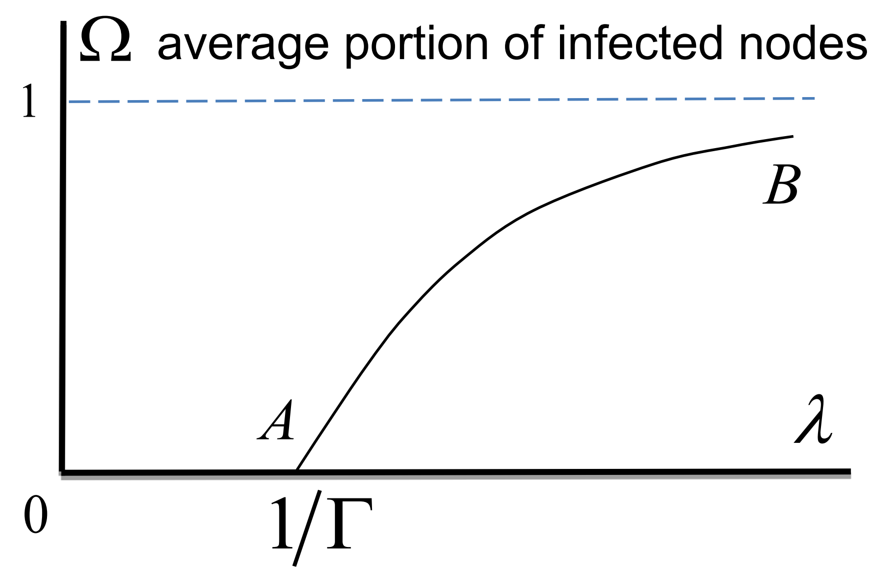
Γ Perron-Frobenius eigenvalue of matrix

$$B = (A_{ij} / \mu_i)_{i,j=1}^{N}$$

For uncorrelated random network with $\mu_i = \mu$:

$$\Gamma \approx \mu^{-1} \max\{d^{\max}, \langle d^2 \rangle / \langle d \rangle\}$$

where d_i node i degree

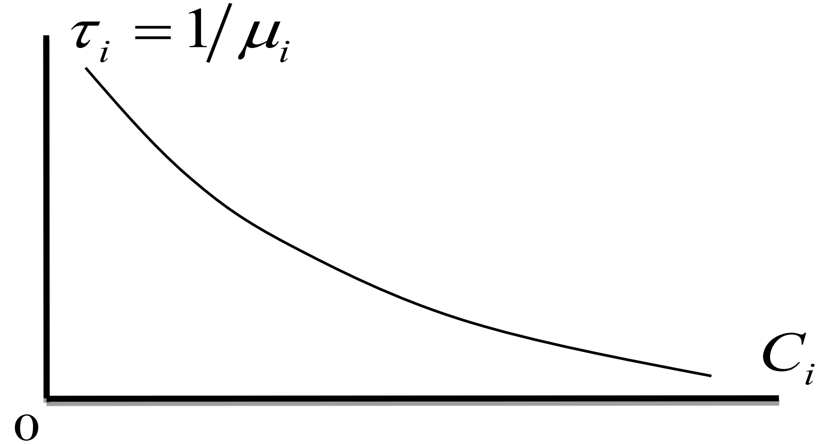


Economics of SIS Infection, given Topology

Node i loss:

$$Loss_i(C) = H_i P_i(C) + C_i$$

where $P_i(C)$ is node infection probability



Average node recovery time vs. node investment

Socially optimal investments:

$$C^{opt} = \arg\min_{C_i \geq 0} \sum_i [H_i P_i(C) + C_i]$$

Selfishly optimal investments: game theoretic framework: Nash equilibrium:

$$C_i^* = \arg\min_{C_i \geq 0} [H_i P_i(C_i, C_{-i}^*) + C_i]$$

Price of Anarchy: $$PoA(C^* | C^{opt}) := \frac{\sum_i [H_i P_i(C^*) + C_i^*]}{\sum_i [H_i P_i(C^{opt}) + C_i^{opt}]}$$

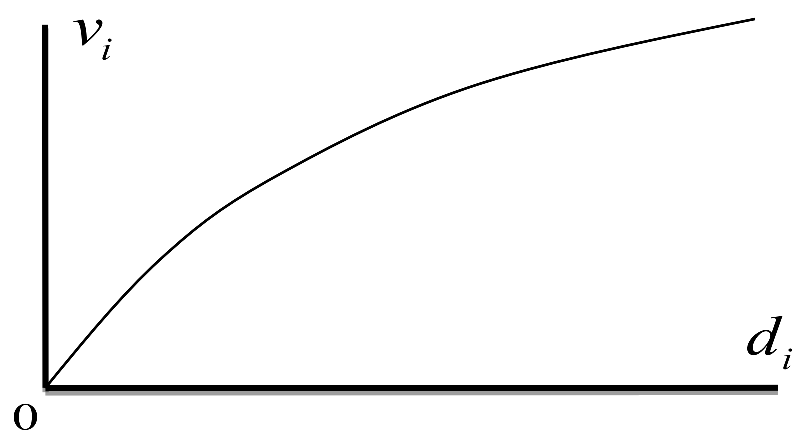
Economics of SIS Infection & Topology

Instantaneous node utility

$$U_i(C,\delta_i \,|\, A) = (1 - H_i\delta_i)v(d_i) - C_i$$

where $d_i = \sum_{j \neq i} A_{ij}, \quad H_i > 1.$

Assumption: network evolves much slower than SIS infection develops



Node utility of connectivity

Averaged over fast time scale node utility

$$\bar{U}_i(C \,|\, A) = [1 - H_i P_i(C \,|\, A)]v(d_i) - C_i$$

Social network optimization: $A^{opt}(C) = \arg\max_A \sum_i \bar{U}_i(C \,|\, A)$

Selfish network optimization by node i: game theoretic framework: Nash equilibrium:

$$A^*_{ij}(C) = \arg\max_{A_{ij}} \sum_i \bar{U}_i(C \,|\, A)$$

Optimal network topology depends on investments C, e.g., socially optimal or selfish

SIS Infection on Growing Network

Consider growing network subject to SIS without investments, where node recovery rate and infection loss depend on the node degree: $\mu_i = \nu_{d_i}$, $H_i = h_{d_i}$

Consider selfish network formation, where arriving node utility of connecting to an existing node depends on this node degree d and infection status δ :

$$u_d(\delta) = (1 - h_d \delta) \log \varphi(d)$$

where $h > 1$, preference for connectivity characterized by $v_i = \log \varphi(d_i)$

Consider logit attachment probabilities to node i: $\alpha_i \sim \exp[T^{-1} u_{d_i}(p_i)]$

$T \to 0$, $T \to \infty$ correspond to complete rationality, randomness

$0 < T < \infty$ describes bounded rationality

Consider $\varphi(d) = d^\beta$, $\beta > 0$: $v_i = \beta \log d_i$, $\alpha_i \sim d_i^{(\beta/T)(1 - h_{d_i} \delta_i)}$

If $h_{d_i} >> 1$, $$\alpha_i \sim \begin{cases} d_i^{\beta/T} & if \quad \delta_i = 0 \\ 0 & if \quad \delta_i = 1 \end{cases}$$

Preferential Attachment under SIS Infection

Case $h_d = 0$ considered in [P. L. Krapivsky, et al, Connectivity of Growing Random Networks, 2000]

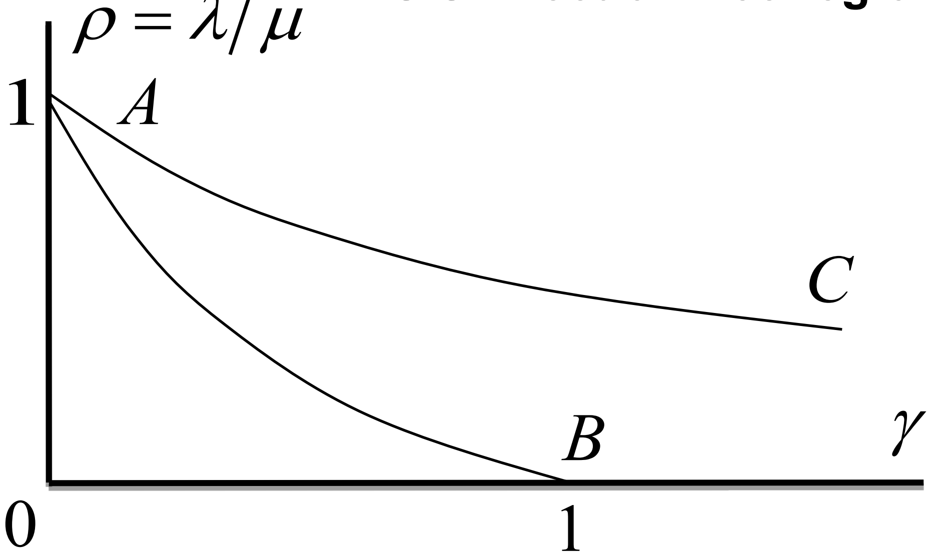
Different behaviors arise for $\gamma < 1$, $\gamma > 1$, and $\gamma = 1$, where $\gamma = \beta / T$

$\gamma < 1$ stretched exponential node degree distribution

$\gamma > 1$ almost a star (winner gets almost all)

$\gamma = 1$ power law with exponent between two and infinity

SIS infection free region



Without infection avoidance

$$h = 0 \rightarrow 0AB$$

With strong infection avoidance

$$h >> 1 \rightarrow 0AC$$

Conjecture: network formation/rewiring may counteract inefficiencies of selfish investments due to externalities

Thank You!